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THEORETICAL MODEL FOR TURBULENT TRANSFER IN THREE-DIMEN-
SIONAL FLUID FLOW

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The aim of this paper is to develop methods of calcu-
lating the velocity and temperature fields for turbulent
flows in the channels of arbitrary shape.

§I. Equations of turbulent motion for averaged values

Let us write the system of equations of turbulent mo-
tion of an incompressible fluid for averaged values in Car-
tesian coordinates.

Equations of motion

$$\frac{\partial \rho \bar{v}_\kappa}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\rho \bar{v}_i \cdot \bar{v}_\kappa \right) = -\frac{\partial \bar{\rho}}{\partial x_\kappa} + \rho \nu \Delta \bar{v}_\kappa - \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\rho \overline{v_i' v_\kappa'} \right) \quad (I.I)$$

the equation of conservation of mass

$$\sum_{\kappa=1}^{3} \frac{\partial}{\partial x_\kappa} \left(\rho \bar{v}_\kappa \right) = 0, \quad (I.2)$$

the equation of heat inflow

$$\frac{\partial \rho \bar{T}}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\rho \bar{v}_i \bar{T} \right) = \rho \kappa \Delta \bar{T} - \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\rho \overline{v_i' T'} \right). \quad (I.3)$$

Here v_1, v_2, v_3 - are velocity components, ρ is the pre-
ssure, T is the temperature, ρ is the fluid density,
ν is kinematic viscosity and k is the coefficient of
temperature conductivity. The symbol Δf means the three-
dimensional Laplace operator. For the sake of simplicity
of writing down the equations (I.3)-(I.4) the coefficient
of viscosity $\mu = \rho \nu$ and the coefficient of heat conductivity
$\lambda = c_j \rho \kappa$, are assumed to be constant.

After the appearance of the Reynolds work [I] in
which the equations (I.I)-(I.2) have been obtained, the
further development of turbulent transfer theory procceded
by way of building of semiempirical models, the basis of
which was the concept of a "mixing" length" similar to one

of the mean free path of the molecules in the kinetic the-
ory of gases (see, for example, the survey of the mono-
graph $[2]$).

In the analysis of one-dimensional fluid flow $w(x)$
the Prandtl-Karman relation is widely used in practice as
a semiempirical approximation for the turbulent stresses
$\rho \overline{u'w'}$ which has a form

$$-\rho \overline{u'w'} = \rho \varepsilon \frac{\partial w}{\partial x} \qquad \text{where} \quad \varepsilon = \ell^2 \left/ \left| \frac{dw}{dx} \right| \right. , \qquad (I.4)$$

ℓ being a turbulent scale ("mixing length").

If one takes $\ell = \varkappa x$ and an appropriate origin for x,
Prandtl relation for the coefficient ε ensured in many
cases a quite satisfactory solution of the hydrodynamical
problems.

The statistical theory of turbulence was developed
later (in twenties-thirties). In spite of the serious the-
oretical successes the statistical theory could not give
a practically acceptable procedure for completing the Rey-
nolds equations.

In this paper an extension of the semiempirical
Prandtl-Karman theory is made to the case of an arbitrary
three-dimenthional fluid flow.

§2. Three-dimensional model ' turbulent transfer
in arbitrary fluid flow

We shall consider the turbulent motion as a super-
position of irregular transient vorticities on the main
averaged motion. Every eddy which appears by chance and
then quickly disappears and which has a diameter equal,
say, to 2ℓ , transfers fluid lumps of ℓ size over the dis-
tances also approximately equal to ℓ .

For the sake of convenience of mathematical descrip-
tion of the momentum and heat transfer caused by random
motions of fluid lumps inside of transient vorticities
we shall suppose that fluid lumps fly out from the vici-
nity of every flow point M condidered in the coordinate
system that moves with the averaged flow velocity. This
flying-out takes place in all directions with equal pro-

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bability. Introduce the notion of a linear scale of turbulence $L(M)$ which characterizes the linear size of transient vorticities in the vicinity of a current point M of fluid flow. We shall suppose that the characteristic "diameter" d of lumps which fly out from the vicinity of point M is identical to the scale L with an accuracy of a constant factor β :

$$d = \beta L . \tag{2.I}$$

Following the idea of the local similarity we shall suppose that the modulus of turbulent velocity of the lump occuring in the vicinity of M is proportional to the modulus of deformation of the velocity field of the averaged motion in M and the characteristic scale L in this point:

$$v'' = \begin{cases} \mu L \left| \dfrac{\partial v}{\partial n} \right| , & \text{if } \dfrac{L^2}{v} \left| \dfrac{\partial v}{\partial n} \right| \geq w , \tag{2.2} \\[2ex] 0 , & \text{if } \dfrac{L^2}{v} \left| \dfrac{\partial v}{\partial n} \right| < w , \tag{2.3} \end{cases}$$

where

$$\left| \frac{\partial v}{\partial n} \right|^2 = 2 \left(\frac{\partial \bar{u}}{\partial x} \right)^2 + 2 \left(\frac{\partial \bar{v}}{\partial y} \right)^2 + 2 \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right)^2 + \left(\frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2 ;$$

μ and ω are dimensionaless constants.[*]
Approximate formula for characteristic linear scale L will be done later.

Now we shall analyse the turbulent stresses $\tau_{ik} = -\rho \overline{v_i' v_k'}$ and the turbulent heat flows $q = c \rho \overline{v' T'}$ in equations(I.I-I.3). In every point M the quantities $\rho \overline{v_i' v_k'}$ and $c \rho \overline{v_i' T_*'}$ are respectively averaged fluxes of the momentum mv and of the heat cmT generated by the turbulent velocity component v_i' in the positive direction of the axis x through the unit

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[*] "Energy equation" (2.2) is similar in its structure to the expression for the kinetic energy dissipation per unit mass in a laminar fluid flow.

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surface perpendicular to x_i and moving with the averaged
flow velocity at M .

Consider a unit area at some point M inside the
flow perpendicular to the x_i-direction. This surface is
considered to move with a velocity of the averaged motion
at M (Fig.I). Lumps from the vicinities of various flow
points can pass through this surface. At some moment t let
lump occuring in the vicinity of M pass the point M_0 with
the velocity V . If the lump moves from M to M_0 without
deceleration, its velocity V would be exactly equal to an
algebraic sum of the averaged motion velocity $\vec{V}$ at M and
the turbulence lump velocity V' taken in the coordinate
system moving with the averaged flow at M . The turbulence
velocity $v_i'(M_0)$ at the moment of passing of the lump would
be accordingly equal to

$$ v_i'(M_0) = V'(M) \cos(S, x_i) + \left[\bar{v_i}(M) - v_i(M_0) \right], \qquad (2.4) $$

where S' is the direction of velocity vector V' in the
coordinate system moving with the averaged flow at M (Fig.I).

But in reality because of the interaction of the lump
with the surroundings lump velocity in the process of its
motion and therefore real turbulence velocity $v_i'(M_0)$ will be
differed from that calculated from (2.4). Analogically at
the moment of passing of the lump through M_0 the tempera-
ture of the lump considered will not be equal to its tempe-
rature at the origin M .

Introduce a hypothesis about the interaction of a
moving lump with the surrounding fluid. The equation for the
momentum and heat flow variation in a moving volume are writ-
ten in the following manner:

$$ dv_i^* = \frac{3}{R} A_1 (\bar{v_i} - v_i^*) dt, \quad dT^* = \frac{3}{R} A_2 (\bar{T} - T^*) dt \qquad \text{x)} \qquad (2.5) $$

Here v_i^* and T^* are velocity components and temperature
in the moving volume $\bar{v_i}$ and T are those in the surrounding
fluid, R is a "radius" of the lump, associated with the
scale $L(M)$ by the relation (2.I), A_1 and A_2 are some coef-
ficients of m/sec dimensions.

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Make in (2.5) the transition from the independent variable t to the variable τ representing a distance between the moving lump and the origin , and it is in the system moving with the mean fluid flow velocity at M :

$$dt = \frac{d\tau}{V},$$

Then equations (2.5) take the form

$$\frac{dv_i^*}{d\tau} + P_1 \, v_i^* = P_1 \, \bar{v}_i \qquad \frac{dt^*}{d\tau} + P_2 T^* = P_2 \bar{T} , \qquad (2.6)$$

where

$$P_1 = \frac{3 A_1}{RV'} , \qquad P_2 = \frac{3 A_2}{RV'} . \qquad (2.7)$$

Supposing the coefficients in (2.6) to be constant and the lump trajectory from M to M_0 to be rectilinear one can find approximate solution of equations (2.6).

In particular, the solution for $v_i^*(\tau)$ at $\tau = S$ where $S = M M_0$ is written as follows

$$v_i^*(s) = e^{-P_1 S} \left[\int_0^s P_1 \, \bar{v}_i \, e^{P_1 \tau'} d\tau' + v_i^*(0) \right] , \qquad (2.8)$$

where

$$v_i^*(0) = V'(M) \cos (S_i' x_i) + \bar{v}_i (M) .$$

Write the velocity $\bar{v}_i$ in the surrounding fluid on the way $M M_0$ as a linear function of the distance τ . Then from (2.8) one can find

$$v_i'(M_0) = v_i^*(s) - \bar{v}_i(s) = V'(M) f_0 \cos (S_i' x_i) + [\bar{v}_i(M) - \bar{v}_i(M_0)] f \qquad (2.9)$$

where $f_0 = e^{-P_1 S}$, $f_1 = \frac{1}{P_1 S}(1 - e^{-P_1 S})$.

Analogically we obtain an expression for the temperature fluctuation $T'(M_0)$ at the moment of passing of the lump considered through the point M.

$$T'(M_0) = [\bar{T}(M) - T(M_0)] f_2 , \qquad (2.10)$$

where $f_2 = \frac{1}{P_2 S}(1 - e^{-P_2 S}) = f_2(P_2 S)$.

We shall suppose that the difference of the averaged flow

The introduction of equation (2.5) is an extension of ideas about an interaction between the moving lump and the surrounding fluid, developed in [7] , [8] , etc.

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velocities $\vec{V}(M) - \vec{V}(M_0)$ at the distances MM_0 of the order
of the mean free path of the lump is much less than the
absolute value of the turbulence velocity V' of lump

When we use the expressions (2.9-2.10) we write the
expressions for the flows $\rho V'(M_0) \, v'_n(M_0)$,

$$\rho v'_i(M_0) \times \overline{V}'(M_0).$$

which are formed at the moment of passing of the lump
from the vicinity of M through M_0 as follows

$$\rho v'_i(M_0) v'_k(M_0) = \mathcal{F}_{ik}(MM_0) \simeq \rho v'^2(M) f_0^2 \cos(s, x_i) \cos(s, x_k) +$$

$$+ \rho V'(M) f_0 f_1 \left[\overline{v}_k(M) - \overline{v}_k(M_0) \right] \cos(s, x_i) + \tag{2.11}$$

$$+ \rho V'(M) f_0 f_1 \left[\overline{v}_i(M) - \overline{v}_i(M_0) \right] \cos(s, x_k),$$

$$\rho v'_i(M_0) T'(M_0) = E_i(M, M_0) \simeq \tag{2.12}$$

$$\simeq \rho V'(M) f_0 f_1 \left[\overline{T}(M) - \overline{T}(M_0) \right] \cos(s, x_i).$$

In the expressions (2.12-2.12) the terms
$\left[\overline{v}_i(M) - \overline{v}_i(M_0) \right]\left[\overline{v}_k(M) - \overline{v}_k(M_0) \right] f^2$ and $\left[\overline{v}_i(M) - \overline{v}_i(M_0) \right]\left[T(M) - T(M_0) \right] f_1 f_2$.
are omitted as "small values of the higher order" and $\cos(s, y_i)$
is assumed to be equal to $\cos(s, x_j)$.

To obtain the averaged values $\overline{\rho v'_i v'_k}$ and $\overline{\rho v'_i T'}$ at M_0
it is necessary to perform the integration of the right-hand
parts of (2.11-2.12) over the surronding volume $\mathcal{D}$ with
the appropriate weight function $f(M \to M_0)$ which is a probability
of passing through the M_0 of the lump whose centre is
out of the unit vicinity of an arbitrary point M .

$$\overline{\rho v'_i v'_k}(M_0) = \int_{\mathcal{D}} \mathcal{F}_{ik}(M, M_0) f(M \to M_0) \, d\tau, \tag{2.13}$$

$$\overline{\rho v'_i T'}(M_0) = \int_x E_i(M, M_0) f(M \to M_0) \, d\tau. \tag{2.14}$$

We shall assume that the motion direction spectrum of the
lumps passing throug the M_0 is approximately isotropic in
the coordinate system moving with the velocity of averaged
fluid flow at M_0.

We shall assume that the probability density $f(M \to M_0)$
has a form of spherically symmetrical normal distribution

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law with the dispersion σ proportional to the turbulence scale L at M_0

$$f(M \to M_0) = \frac{1}{4\pi S^2} \sqrt{\frac{2}{\pi}} \frac{1}{\alpha L_0} e^{-\frac{1}{2} \cdot \frac{S^2}{(\alpha L_0)^2}}, \qquad (2.15)$$

where L_0 is the value of the scale L at M_0 and α is a dimensionless constant.

In the first approximation we shall suppose that the turbulent scale $L(M)$ in (2.1) and (2.15) is identical to the characteristic distance between M and the channel walls and determined by

$$\frac{1}{L} = \frac{1}{\pi} \int_{\Omega} \frac{e}{\ell} \, d\omega \qquad (2.16)$$

where ℓ is the distance between M and the channel wall in the direction ω.

In the second approximation the scale $L(M)$ is assumed in addition to be dependent on local peculiarities of the velocity field of the averaged flow [4].

In the case of the forced fluid flows in closed channels it is assumed that the approximation (2.16) for the scale L is sufficient. The form of the function calculated from (2.16) for the channels of various sections is shown in §4. At this place we note that factor scale values L obtained from (2.16) are apart from a constant in good agreement with the experimental estimates of turbulence scales ([5], [6] etc.)

Thus, all the functions in the integral expressions are obtained. The expressions (2.23) and (2.14) are general integral formulas for the turbulent stresses tensor components and the turbulent flow vector components.

To simplify the use of the obtained approximations for $\overline{\rho v_i' v_k'}$ and $\overline{\rho v_i' T'}$ make some further transformations for the expressions (2.13),(2.14).

The difference of v_i and of T at the points M and M_0 is represented as an expansion along the coordinate axis.

$$\psi(M) - \psi(M_0) \simeq -\left(\frac{\partial \psi}{\partial s}\right)_0 \cdot S = \sum_{i=1}^{3} \left(\frac{\partial \psi}{\partial x_i}\right)_0 \cdot S \cdot \cos(s, x_i) \qquad (2.17)$$

where $\left(\frac{\partial \psi}{\partial s}\right)_0$ is a value of $\frac{\partial \psi}{\partial s}$ at M_0.

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Taking into account the expressions (2.2),(2.II),
(2.I2) and (2.I7) the equations(2.I3),(2.I4) for $\rho \overline{v_i' v_k'}$ and
$\rho \overline{v_i' T'}$ is written in the form

$$-\rho \overline{v_i' v_k'} = -\rho \mathcal{P}_{ik} + \sum_{j=1}^{3} \rho \varepsilon_M^{ij} \frac{\partial \overline{v_k}}{\partial x_j} + \sum_{j=1}^{3} \rho v_M^{kj} \frac{\partial \overline{v_i}}{\partial x_j} \qquad (2.18)$$

$$\rho \overline{v_i' T'} = \sum_{j=1}^{3} \rho \varepsilon_M^{ij} \frac{\partial \overline{T}}{\partial x} ; \qquad (2.19)$$

where

$$\mathcal{P}_{ik}(M_0) = \mu_0^2 \int L^2 \left| \frac{\partial v}{\partial n} \right|^2 f_0^2 f(M \to M_0) \cos(s, x_i) \cos(s, x_k) d\overline{\tau}, \qquad (2.20)$$

$$\varepsilon_M^{ej}(M_0) = \mu \int L \left| \frac{\partial v}{\partial n} \right| f_0 f_1 \, s f(M \to M_0) \cos(s, x_e) \cos(s, x_j) d\overline{\tau}, \qquad (2.21)$$

$$\varepsilon_M^{ej}(M_0) = \mu \int L \left| \frac{\partial v}{\partial n} \right| f_0 f_2 \, s \cdot f(M \to M_0) \cos(s, x_e) \cos(s, x_j) d\overline{\tau}. \qquad (2.22)$$

The summands in (2.I8) have the same structure as
Prandtl's stress in the case of plane flows. And the terms
of type $\mathcal{P}_{ik}$ are a turbulent analog of the statical pressure
in the fluid·   In the case of calculation of practically
important problems the system of the equations (I.I-I.3) and
(2.I8-2.I9) can in every specific case be considerably simp-
lified if we take into account known hydrodynamical features
of the fluid flow considered and the needed accuracy for the
solution·

In particular, far from the entrance section the equa-
tions of motion (I.I) and of heat inflow for turbulent
fluid flows in rectilinear channels   be written after
using (2.I8-2.I9) and omitting unessential terms.

$$O = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{\partial}{\partial x} \left\{ \left(v + \varepsilon_M'' \right) \frac{\partial w}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ \left(v + \varepsilon_M^{22} \right) \frac{\partial w}{\partial y} \right\}, \qquad (2.23)$$

$$w \frac{\partial T}{\partial z} = \frac{\partial}{\partial x} \left\{ \left(k + \varepsilon_M'' \right) \frac{\partial T}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ \left(k + \varepsilon_M^{22} \left| \frac{\partial T}{\partial y} \right| \right) \right\} + \frac{\partial}{\partial z} \left\{ \left(k + \varepsilon_M^{33} \right) \frac{\partial T}{\partial z} \right\}, \qquad (2.24)$$

where

$$\varepsilon_M^{jj}(M_0) = \mu \int L \left| \frac{\partial w}{\partial n} \right| f_0 f_1 \, s f(M \to M_0) \cos^2(s, x_j) d\overline{\tau}, \qquad (2.25)$$

$$\varepsilon_M^{jj}(M_0) = \mu \int L \left| \frac{\partial w}{\partial n} \right| f_0 f_2 \, s f(M \to M_0) \cos^2(s, x_j) d\overline{\tau}, \qquad (2.26)$$

and

$$\left| \frac{\partial w}{\partial n} \right| = \sqrt{\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2}$$

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§3. Values of the coefficients μ, α, A_1 and A_2

Determine the empirical coefficients α, μ, A_1 and A_2 included in the discussed model of turbulent transfer by means of some experimental measurements of the correlation moments in turbulent flow and by using other empirical data.

First of all to ensure the agreement between the empirical data and the results of calculations according to (2.18) of the root-mean-square turbulent velocity and the tangential stresses in turbulent fluid flows we must assume

$$\mu = 1,7 \div 1,8 \; , \qquad \mu/\alpha = 0,75. \qquad (3.1)$$

The mechanism of the interaction between a lump and the surroundings, described by (2.5) is assumed to be double. Firstly, due to the molecular diffusion the moving lump exchanges the quantities v_i and T with the surroundings. Secondly, the moving lump which is not a solid but some "seething fluid lump" exchanges with the surrounding fluid by "macro-particles".

Therefore, represent the coefficients A_1 and A_2 involved in (2.5) as a sum of two terms corresponding to the action of a molecular mechanism of quantity transfer and to the transfer by "macro-particles":

$$A_1 = \left(b_1 + b_2 \right) \frac{\nu}{R} \; , \qquad A_2 = b_3 \frac{\kappa}{R} + b_4 \frac{\nu}{R} \; , \qquad (3.2)$$

where b_1, b_2, b_3, b_4 are dimensionaless quantities and b_1, b_3 are nearly unities.

Following [4] one can adopt

$$b_4 = b_2 \; , \qquad b_3 = b_1 \left(\frac{\nu}{\kappa} \right)^{0,33} \qquad (3.3)$$

Taking into account (3.2-3.3) and (2.2) one may write the arguments of functions f_0, f_1 and f_2 in the form

$$P_1 S = \frac{12 \theta^2 \left(b_1 + b_2 \right)}{\mu \alpha \cdot \gamma_*} \cdot \frac{S}{\alpha L} \; , \qquad P_2 S = \frac{12 \theta^2 \left(b_1 \mathcal{D}^{0,67} + b_2 \right)}{\mu \alpha \gamma_*} \frac{S}{\alpha L} \; , \quad (3.4)$$

where

$$\gamma_* = \frac{L^2}{\nu} \left| \frac{\partial v}{\partial n} \right| \; , \quad \mathcal{D} = \frac{\kappa}{\nu} \; , \qquad \theta = \frac{\alpha}{\beta}$$

For the sake of simplicity we shall assume   and

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to be constant. The coefficient θ which is a factor for b_1 and for b_2 may be introduced into b_1 and b_2 so that we shall assume it to be equal unity.

Following [4] take

$$b_1 = 1 , \qquad b_2 = 4. \tag{3.5}$$

After some trial calculations of velocity and temperature fields for the fluid flows in a round pipe the values of the coefficients μ, α, b_1 and b_2 finally have been taken

$$\mu = 1,8 ; \quad \alpha = 0,42 ; \quad b_1 = 0,9 ; \quad b_2 = 3,8. \tag{3.6}$$

The critical number ω has been taken to be equal to 25.

§4. Results of calculations of the velocity and temperature fields for turbulent fluid flows in rectilinear channels

The developed model of turbulent transfer is used for calculating the velocity and temperature fields in turbulent flows in a circular tube, in ring and flat clearences, in channels of rectangular section and in cells of rod lattice.

In these problems in practical solving the equations of type (2.23) and (2.24) it is used some simplifications of the expressions (2.25) and (2.26) for the coefficients ε_M^{jj} and ε_H^{jj}. So the integral expressions for ε_M^{jj} and ε_H^{jj} over a lateral region $\mathcal{D}$ around some considered point are simplified to integrals over the segment parallel to the axis x_j.

$$\varepsilon_M^{jj}(M_0) = c L_0 \int_{-\infty}^{\infty} L \left| \frac{\partial w}{\partial n} \right| f_0(q \varepsilon_j) f_1(q \varepsilon_j) G(\varepsilon_j) d\zeta_j , \tag{4.1}$$

$$\varepsilon_H^{jj}(M_0) = c L_0 \int L \left(\frac{\partial w}{\partial n} \right) f_0(q \zeta_j) f_1(A q \zeta_j) G(\zeta_j) d\zeta_j , \tag{4.2}$$

where $\zeta_j = \dfrac{x_j - \ldots}{\alpha L_0}$, $G(\zeta) = \frac{1}{2}|\zeta| e^{-1/2 \zeta^2}$, $q = \alpha L_0 P_1$.

$$\frac{u\alpha}{3}, \quad A = 0,8 + 0,2 \mathcal{D}^{0,67} \tag{4.3}$$

The function $L(\mu)$ in sections in infinite rectilinear channels was calculated according to the formula (2.16) which for such channels is reduced to

$$\frac{1}{L(\mu)} = \frac{1}{2} \int_0^{2\pi} \frac{1}{\ell} df \tag{4.4}$$

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where $\ell(f)$ is the distance between the point M and perimeter of the section of the channel in the direction f .

For the 2ℓ -wide channel (or $2b$ wide annulus) between two parallel plates the formula (4.4) gives (see [4])

$$\frac{L}{b} = \left(1 - \frac{z}{2} \right) , \quad z = \frac{y}{b} , \tag{4.5}$$

y is a distance between M and one of the plates. According to (4.4) it is also easy to obtain a formula for the scale $L(M)$ in a section of a tube of radius a :

$$\frac{L}{a} = \frac{1 - \zeta^2}{2 E (\zeta, \pi/2)} ,$$

where

$$\zeta = \frac{r}{a} , \quad E(\zeta, f) = \int_0^\ell \sqrt{1 - \zeta^2 \sin^2 \alpha} \, d\alpha \tag{4.6}$$

A curve for the function $L(\zeta)$ for a round tube is given in Fig.2 . It is seen from Fig.2 that the mixing length ℓ calculated by Nikuradse [5] according to the measured velocity profiles in a tube is practically identical (with the accuracy to the constant factor) to the scale L : $\ell = 0,44 L$.

The quantity $\tilde{a}$ is defined for an arbitrary channel by $\frac{1}{\tilde{a}} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\ell_c} df = \frac{1}{\pi \ell_c}$. $\tag{4.7}$
where $\ell_c(f)$ is a distance between the centre"channel" and the wall in the direction f is called as the effective channel radius and in the following will be used as a characteristic cross size of a channel when dimensionless hydrodynamical characteristics are built up .

In particular, for a round tube of the radius a one has according to (4.7): $\tilde{a} = a$; for a flat $2b$ -wide annulus one obtains $\tilde{a} = \frac{\pi}{2} b$; for a channel of rectangular section with the sides $2a$ and $2b$ $(b < a)$ one has

$$\tilde{a} = \frac{\pi}{2} \frac{ab}{\sqrt{a^2 + b^2}} \tag{4.8}$$

Figures 3 and 7 show the results of calculating a mean-over-section dimensionless velocity $\mathcal{U} = \frac{u}{v_*}$ and resistance coefficient $\mathcal{F} = \frac{8}{\bar{v}^2}$ for a round tube when the dimensionless dynamical parameter $\phi = \frac{a v_*}{\nu}$ is varied in a wide range. Here a is the tube radius and $v_*^2 = \frac{a}{r} \frac{1}{\rho} \left| \frac{\partial p}{\partial x} \right|$.

For $\phi < 32$ ($Re < 500$) the solution for $\bar{u}$ turns into

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the solution for the laminar regime $\bar{U} = 0,25\phi$.

Computed velocity profiles U over the tube radius are shown in Fig.4. It is drawn the conventional curves $U = f\left(\frac{yV*}{\nu}\right)$ for different Reynolds number Re, where y is the distance from the wall. The picture of branching of curves $V = f\left(\frac{yV*}{\nu}\right)$ for different Reynolds number R represented in Fig.4 looks like the one corresponding to the experimental data (see, for example, 9)·

Computed profiles of turbulence viscosity $\mathcal{E}\mu$ over tube section at different values of Re are represented in Fig.5.

As it is seen from figures 3-5 the results of velocity field computations for tubes are in a good agreement with the experimental data·

Note that in computing the velocity field (as well as the temperature fields in fluid with $Pr \leq I$) the condition (2.3) is found to be important, however, in computing the temperature fields in fluid flows with $Pr > I$ the condition (2.3) involved decreases the computed number Nu and gives rise to the more strongly pronounced transition of the solution for Nu to a solution for a laminar flow (for Re 400)·

Figure 6 shows the results of computations of the number Nu for flows of different fluids in a round tube when the condition at the tube wall is q =const where q is a heat flux density· In the number Nu intervals from 3.10^3 to 3.10^6 the represented results of computing the number Nu in a round tube may be described by an interpolation formula

$$Nu = A + 3,90 \left(Re \cdot 10^{-3}\right)^m Pr^n,$$ (4.9)

where $\quad A = 2,5 + 1,3 \, lg \left(1 + Pr^{-1}\right),$
$\quad\quad m = 0,918 - 0,051 \, lg \left(1 + 10 Pr^{-1}\right)$
$\quad\quad n = 0,65 - 0,107 \, lg \left(1 + 10 Pr\right)$

The computation results represented in Fig.6 are in rather good agreement with the experimental data both for fluids with $Pr < I($ [IO], [II] etc.) and for the fluids with large number Pr([II], [I2] etc.).

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Fig.7 shows the results of computing the resistance coefficients $\zeta = \dfrac{\mathcal{E}}{U^2}$ for annular gaps formed by two coaxial cylinders with the radii a_1 and a_2 $(a_1 < a_2)$ at different values of the dynamical parameter $\phi = \dfrac{\widehat{a}\, v_*}{\nu}$ (or at different

$Re = \dfrac{2\widehat{a}\, u}{\nu}$)

where $v_*^2 = \dfrac{\widehat{a}}{2} \cdot \dfrac{1}{\rho}\left|\dfrac{\partial P}{\partial z}\right|$, $\quad U = \dfrac{u}{v_*}$,

and

$$\widehat{a} = \left[\frac{\pi}{4} + \left(1 - \frac{\pi}{4}\right)\left(1 - \frac{a_1}{a_2}\right)^2\right](a_2 - a_1) \qquad \text{\textcyrillic{ж})}$$

(4.10)

As it is seen from Fig. 7 the curves for $\zeta = f(Re)$ in annular gaps at different parameters $\theta = \dfrac{a_1}{a_2}$ are between the corresponding curves $\zeta = f(Re)$ for the round tube and for the flat clearance.

When ϕ is decreasing the solution $\overline{U}$ for a flat annulus in chosen variables is approached to the formula $\overline{U} = 0.27\,\phi$

In Fig.9 an example of computing the velocity field $U(x,y)$ in a channel of rectangular section with the parameter $\gamma = \dfrac{a}{b} = 2$ for arbitrary channels $U = \dfrac{u}{v_*}$, $v_*^2 = \dfrac{a}{2} \dfrac{1}{\rho}\left|\dfrac{\partial P}{\partial z}\right|$, $\phi = \dfrac{\widehat{a}\, v_*}{\nu}$, $Re = \dfrac{2\widehat{a}\,\overline{u}}{\nu}$, $\zeta = \dfrac{\mathcal{E}}{U^*}$.

The results of computing the resistance coefficient ζ in turbulent fluid flows in the channels of the rectangular section are represented in Fig.8 at different values of the relation of the section size γ one to another. For a given value of Reynolds number the resistance coefficient ζ decreases with the parameter increasing.

The results of measurements of the resistance coefficients ζ in the channels of rectangular section are represented in Fig8 15 . As follows from Fig.8 the results of the calculations of the resistance coefficients in channels of rectangular section are in rather good agreement with the results of the measurements.

In view of the fact that the temperature field in a fluid flow in a channel of the rectangular section is determined by a great number of parameters such as: the heat

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\textcyrillic{ж} The formula (4.10) is an interpolating one

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source distribution , the design and the heat-conductivity
of channel walls, the number Pr for the fluid and the numbers
Re and j, it is not necessary to construct any generalized
characteristics of the temperature field as functions of the
numbers Re, Pr etc'

It is worth to calculate the temperature fields using
(2.24) only for a given design of heat-exchanger cell.

Give now some results of velocity and temperature
field calculations for turbulent fluid flows in rod lattices.
In Fig.IO an example of the velocity field calculation for
the triangular array with the relative spacing h =I.2 is
given for the case of a longitudinal flowing around of this
array.

The results of the resistance coefficient calculations
for the triangular array as functions of Re and at the re-
lative spacing h are represented in Fig.II.

When the characteristic cross size of the channel $\tilde{a}$
has been taken here, the computed curves $\xi = f(Re, h)$ for diffe-
rent values of relative spacing h are in a logical sequence.
At equal values of the number Re the resistance coefficients
ξ increase with deviating the channel section shape from
the circle one.

When however one uses the hydraulic radius R' as a
defining cross-size of channel the obtained generalized de-
pendence $\xi' = f(Re', h)$ has no such logical regularuty.

Fig.II shows also the resistance coefficient calcu-
lation results for the cell of rectangular lattice of rods
with the relative spacing h =I.0. Here the results of, re-
sistance coefficient measurements for the cells of different

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x) The values of the coefficient ξ for rectangular channels
computed in [I5] using Daysler-Taylor graph-analitical
method are close to the values ξ obtained in this paper.

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—I4—

lattices taken from [I4] and [I3] are shown by the dotted curves. As it is seen from Fig.II, the resistance coefficients computed in this paper for the lattices with the relative lattice spacing h =I.0-I.2 are in satisfactory agreement with the experimental results.

Fig.I2 shows an example of computing the dimensionless temperature $\theta = \dfrac{\lambda m (T - T_c)}{2 q_0 R}$ on surface of the fuel rod in the cell of triangular array with h =I.0 when the fluid with P_τ =0.025 flows through this cell . The number Re is equal to 5.10^4. Here the rod section at the circle with radius τ_0 =0.773.R where R is the rod radius. The problem conditions here correspond to the physical ones for the experiments [I6] . In Fig.I2 the dotted line represents the results of measurements of the function θ from [I6] .

It is seen from Fig.I2 that the results of computing the temperature θ for these specific conditions are in a rather good agreement with the experimental ones.

Thus, the results of calculation of the velocity and temperature fields in the fluid flows in different channels established in this paper allow to think that the later model of turbulent transfer represented may be used for computing the hydraulic resistances and thermal regime in the cells of heat-exchanger of arbitrary design.

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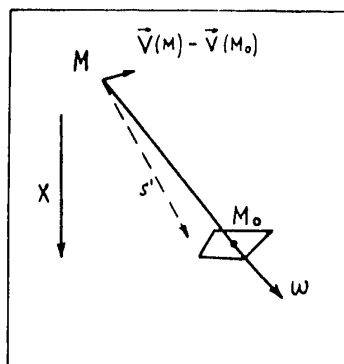
Fig.2 Comparison of the function L and ℓ for the circular pipe
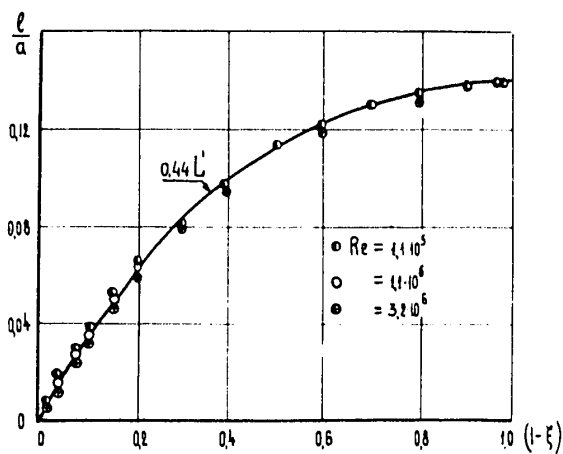
Fig.3 The mean velocity $\bar{U}$ vs the dynamical parameter ϕ for circular pipe 1-compound curve, 2-experimental curve

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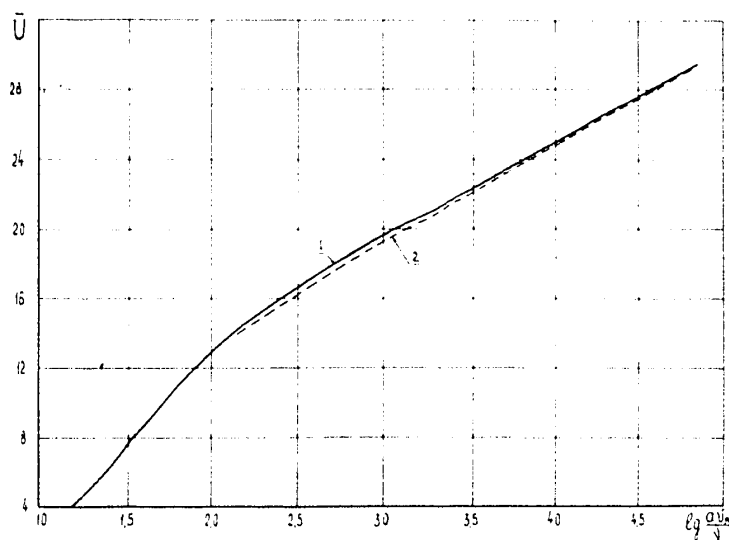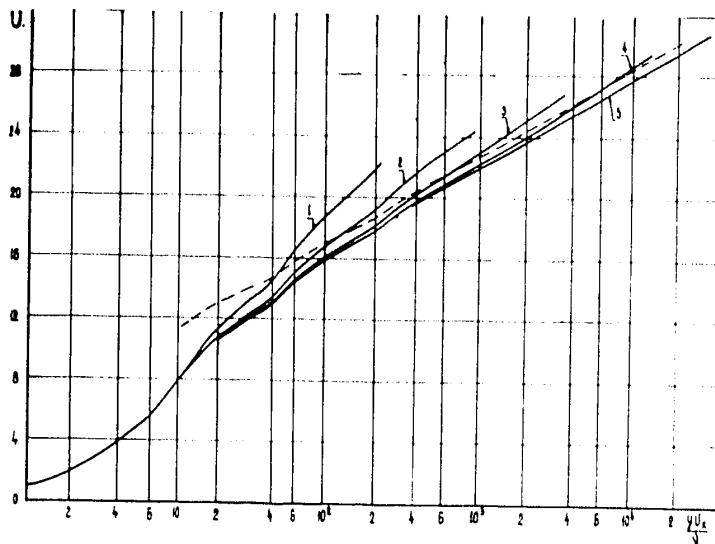
Fig.4 Computed velocity profiles $v(y)$ in a circular pipe at different values of number Re .
I-5 correspond to Re= =6.9.10^3, 3.4.10^4, I.6.10^5, 3.2.10^6.
Dotted curve corresponds to Prandtl solution:

$$v = 5,5 + 5,75 \, \lg \frac{y v_*}{\nu} \, .$$
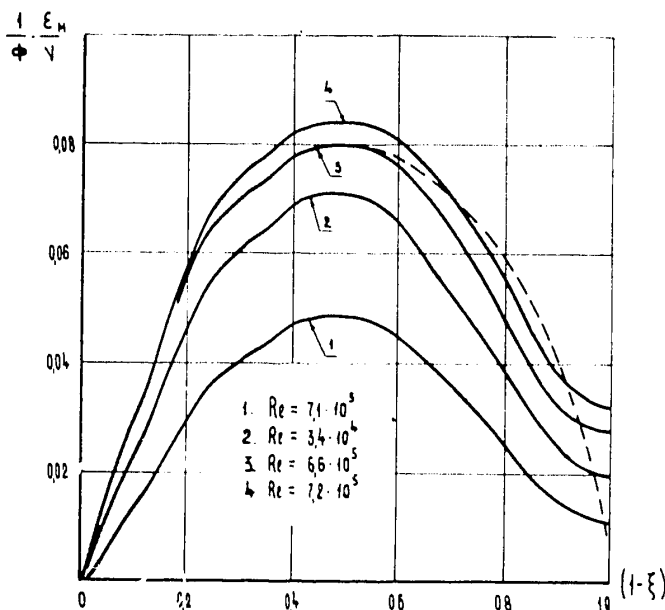
Fig.5 Computed turbulent viscosities ε_H in pipe fluid flow I-4 correspond to Re= $6.9_s 10^3, 3.4.10^4$, I.6.10^5 7.3.10^5.
Dotted curve correspond to the experimental data[5] for Re=10^5- 10^6.
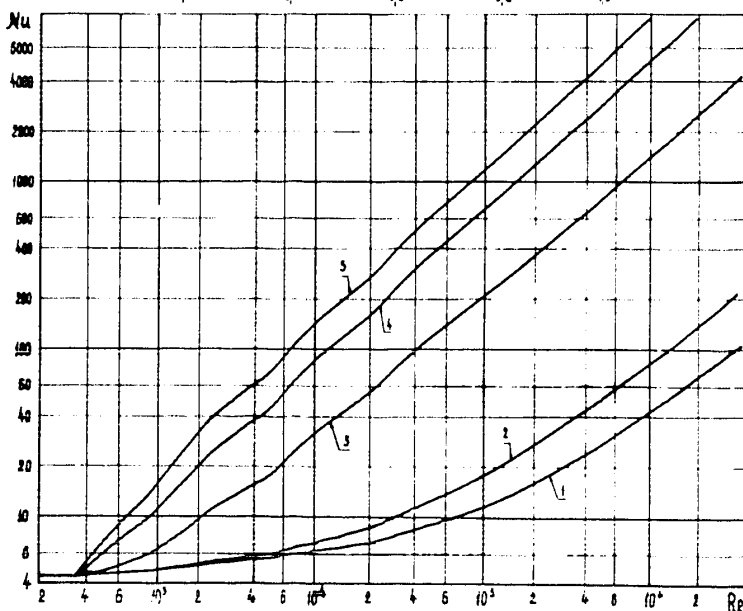
Fig.6 Computed numbers of Nu for fluid flows in circular pipe.
I-Pr=0.0I0,2-Pr=0.025, 3-Pr=I.0, 4-Pr=I0, 5-Pr=I00.
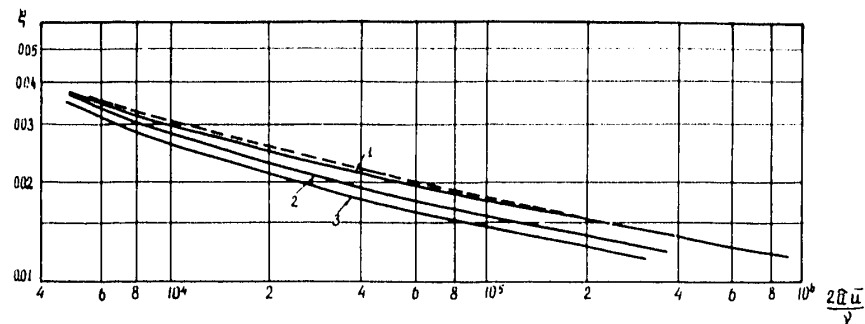
Fig.7 Computed resistance coefficients ξ for ring and flat
clearances. I-circular pipe ($\Theta = 0$) . 2-ring clearance
with Θ =0.5; 3-clearance with Θ =0.8-I.0;
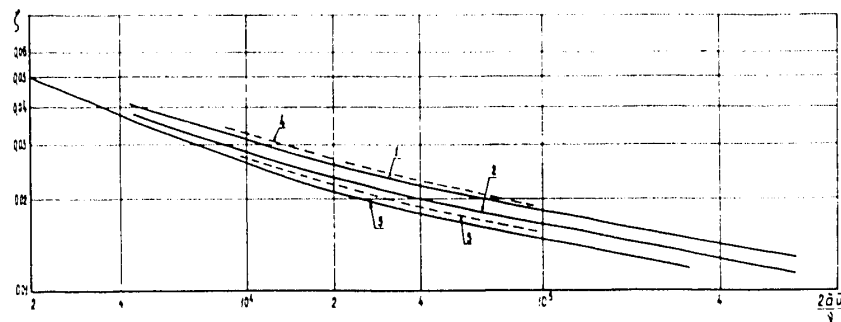− − − − experimental data for circular pipe.

Fig.8 Computed resistance coefficients ξ for the channels of
rectangular section. I-2 correspond to γ =I and γ =5.
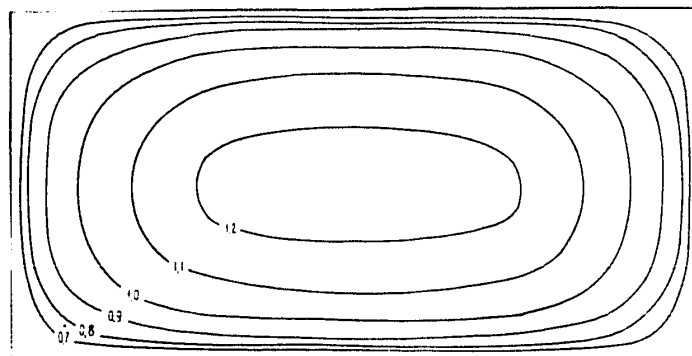flat clearance; 4,5- are the measurement results [I5]
for γ =I and γ =5÷I0.

Fig. 9 Computed velocity field $v(x,y)$ in channels of rectangular
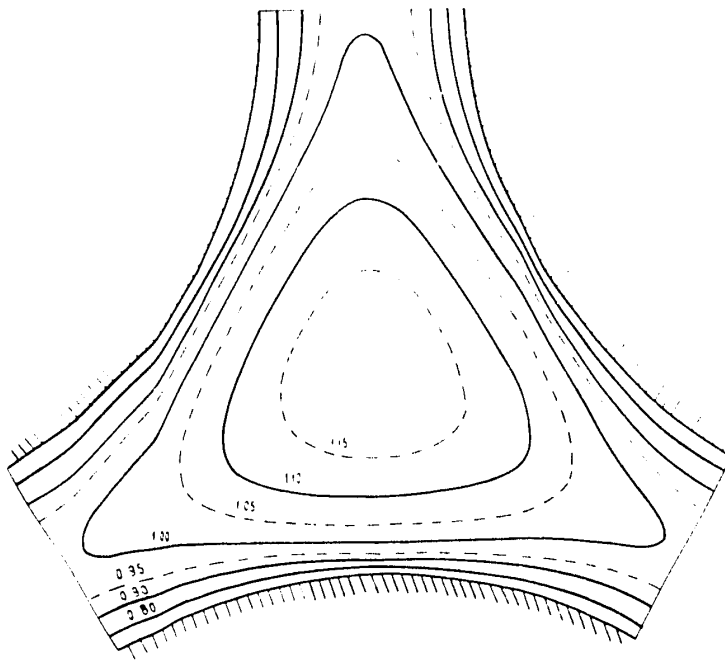section. γ =2, Re=3.4xIO^4.

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Fig.IO Computed veloci-
ty field $v(\tau,\rho)$ for the
cell of triangular ar-
ray of rods with the
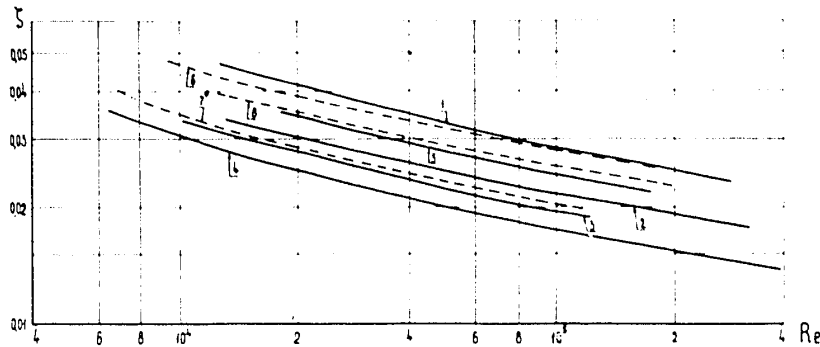relative lattice spa-
cing h =I.2;

Re= $2.48.10^4$.

Fig.II Computed resistance coefficients ξ for the cells of tri-
angular and quadratic array of rods for different lattice
spacing h . I,2,3-triangular array h=I.0; I.I; I.2;
4-circular pipe; 5-the quadratic lattice cell with h=I.O.
Experimental curves. 6,7-triangular array with h=I.0 I4
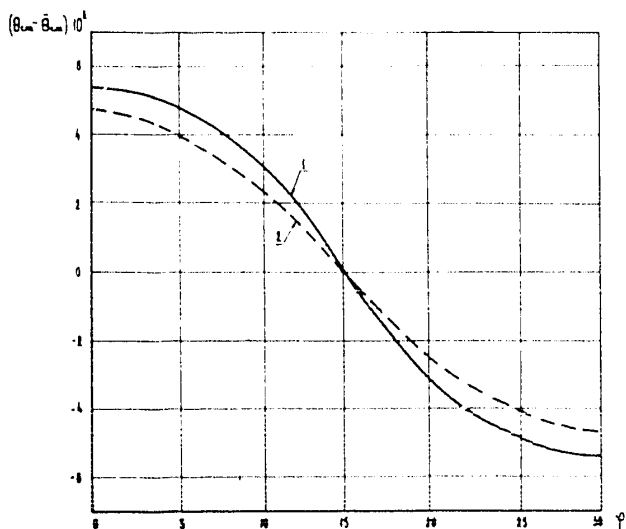and h=I.2 [I3] . 8-quadratic array with h=I.0 [I4] .

Fig.I2 An example of computing
the dimensionless temperature
$\theta = \frac{\lambda_m (T-T_c)}{\tau q_o R}$ at the surface of
fuel rod in the cell of triangle
lattice with the spacing h=I.0;
Pr=0.025; Re=5.10⁴.

The additional explanations are
given in the text;
I-the results of the calculation
2-the results of the measure-
ments [I6].

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